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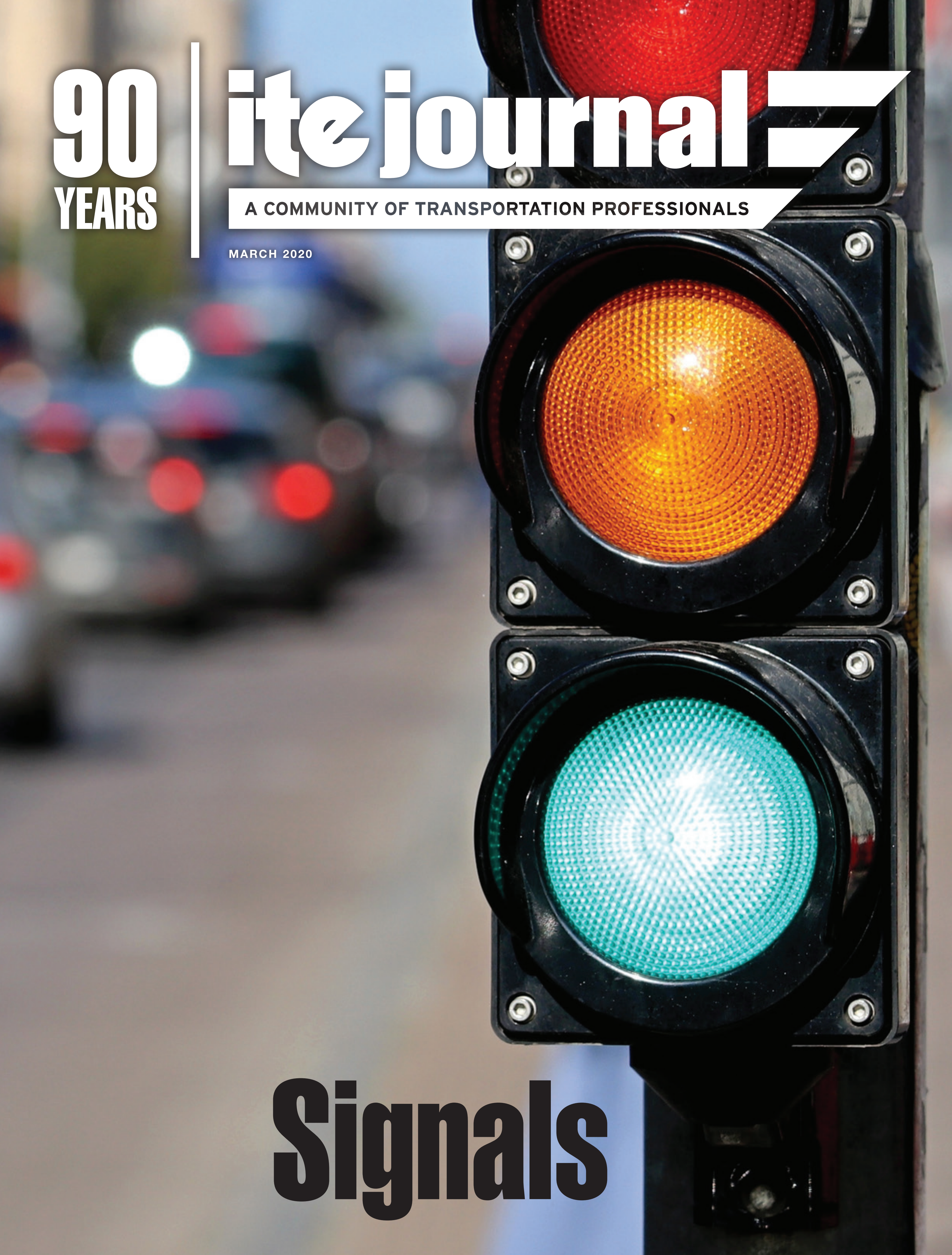

Signals

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An Explanation of Mats Järlström's Extended Kinematic Equation

By Jay Beeber (M)

Since the yellow indication was first added to traffic signals in 1920, the proper interval duration has been robustly debated.[1] Seemingly, the timing of the yellow indication appears straightforward. However, determining the illumination interval is quite intricate since it is part of a complex system of physical and human-made laws, technology, and human behavior that all must operate compatibly.

In 1960, Denos Gazis, Robert Herman, and Alexei A. Maradudin (GHM) provided a scientific solution to the yellow change interval question in their paper, "The Problem of the Amber Signal Light in Traffic Flow."[2] GHM presented a kinematic solution to a binary STOP or GO dilemma when a driver is faced with the onset of a yellow signal indication. The problem GHM solved and eliminated was an area in the roadway known as the "dilemma zone", where a driver-vehicle complex could neither STOP safely and comfortably nor GO without the need to violate the red or accelerate unsafely into the intersection.

GHM's solution to regulate a yellow change interval first appeared in the 1965 ITE *Traffic Engineering Handbook*, and it has become known as the kinematic equation.[3] However, GHM's solution is limited to vehicles traveling through level intersections at constant velocity, which does not include vehicle deceleration to execute safe turning maneuvers. This article presents a brief review covering GHM's original solution and Mats Järlström's extended kinematic equation which allows for vehicle deceleration and turning maneuvers.[4]

GHM's Solution

The foundation of GHM's solution is a minimum safe and comfortable DISTANCE to STOP, defined as the "critical distance" (x_C), which is composed of an allocated perception-reaction distance (x_{PR}) plus a minimum braking distance (x_{Br}). It is expressed mathematically as:

$$x_C = x_{PR} + x_{Br} = v_0 \bullet t_{PR} + \frac{v_0^2}{2a_{max}} \qquad (1)$$

Where:

x_C= Critical distance - the minimum safe and comfortable stopping distance, (feet [ft.] or meters [m])

v_0= Maximum uniform (constant) initial/approach velocity, (foot per second [ft./s] or meter per second [m/s])

t_{PR}= Maximum allocated driver-vehicle perception-reaction time, (s)

a_{max}= Maximum uniform (constant) safe and comfortable deceleration, (ft./s^2 or m/s^2)

GHM's GO solution is the minimum TIME needed for a vehicle to travel across the critical distance (x_C) and is thus the minimum yellow change interval (Y_{min}) required to eliminate the dilemma zone. The solution is calculated by dividing the critical distance by the vehicle's maximum constant velocity across that distance. For driver-vehicles that maintain their initial velocity (v_0) across the critical distance, this is expressed mathematically as:

$$Y_{min} = \frac{x_c}{v_0} = \frac{v_0 t_{PR}}{v_0} + \frac{\frac{v_0^2}{2a_{max}}}{v_0} \qquad (2)$$

Which reduces to the well-known kinematic equation:

$$Y_{min} = t_{PR} + \frac{v_0}{2a_{max}} \qquad (3)$$

Since restrictive yellow laws (drivers must not enter the intersection on yellow) prevailed in their jurisdiction, GHM's original yellow time solution also included the minimum clearance interval (t_{Cl}) to allow a vehicle with length (L) to travel straight through and exit an intersection with a width (w), expressed as:

$$t_{Cl} = \frac{w + L}{v_0} \qquad (4)$$

Internationally, "permissive" yellow change laws (driver-vehicles may enter the intersection during the entire yellow interval) are most common and the clearance interval function is often handled by employing a separate "all-red" interval.

Figure 1 illustrates the above concepts for both restrictive (Y_R) and permissive (Y_p) yellow timing policies.

This article promotes the most common permissive yellow change interval timing policy, but practitioners should note that where restrictive yellow laws prevail, the yellow interval must also handle the clearing function.

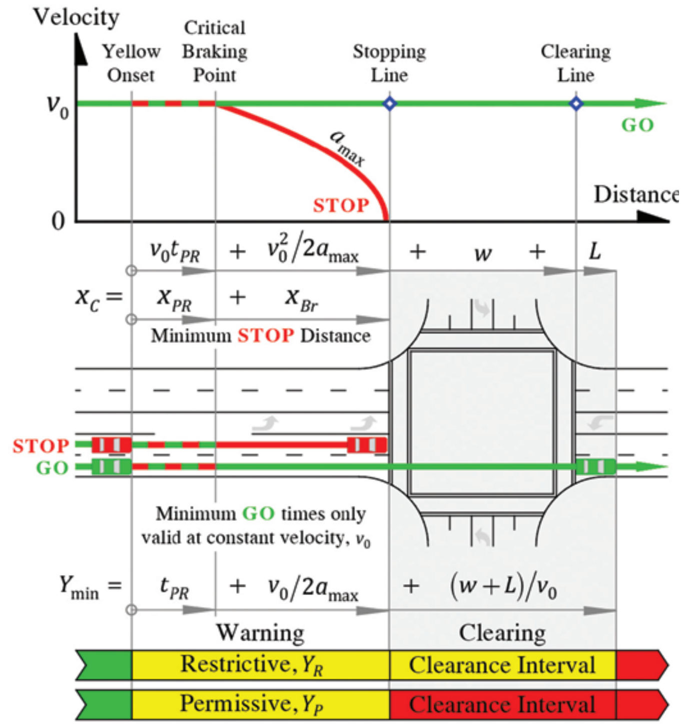

Figure 1. GHM's minimum STOP and GO equations plotted and referenced to a signalized intersection.

Limitations of GHM's Kinematic Equation

An essential concept to be recognized is that GHM's Kinematic Equation can only be derived if both the initial velocity (v_0) which is used to calculate the minimum stopping distance and the vehicle's velocity while traversing the minimum stopping distance are the same. Where a vehicle must slow down for any reason, such as to negotiate a turn, the initial velocity (v_0) and the vehicle's velocity while traversing the critical distance are NOT the same and GHM's Kinematic Equation cannot be used. This point has been reiterated in correspondence by Dr. Alexei A. Maradudin, the sole surviving author of the original GHM paper:[5]

"This formula which we derived, cannot be applied to turning lanes or to any situation where the driver must decelerate within the critical distance. The formula can only be applied to vehicles which start at the maximum allowable speed measured at the critical stopping distance and which proceed at a constant speed into the intersection."

Järlström has devised a new protocol to extend the kinematic equation for situations where a vehicle must slow down within the minimum stopping distance based on GHM's logic.

GHM's Logic Extended to Turning Movements

A central axiom of traffic signal timing is that, at the onset of the yellow indication, a "reasonable" driver farther from the intersection than their minimum stopping distance (critical distance) has sufficient distance to stop comfortably and should do so. Likewise, a "reasonable" driver closer to the intersection than their critical distance proceeds into the intersection when presented with a yellow indication. Figure 2 illustrates this concept.

The logic behind the methodology for determining the duration of the yellow change interval is that the interval should provide a reasonable driver who is too close to the intersection to stop safely and comfortably (i.e., closer than the critical distance) with adequate time to traverse the minimum stopping distance and legally enter the intersection before the signal turns red.

A reasonable driver is defined as one who is not violating the law (i.e., acting legally), and whose chosen actions are rational, prudent, and feasible. Safety and equity requires that the motion of any roadway user who exhibits reasonable behavior must be accommodated within the signal timing protocol, even if their chosen actions are not the "average" or most common to be encountered upon the roadway.

In conformance with the standard for through lane movements, the calculation of the minimum yellow change interval for turning movements must also provide a reasonable driver adequate time to traverse the minimum stopping distance and legally enter the intersection before the onset of the red indication. This calculation must allow for the extra time necessary for a vehicle to traverse

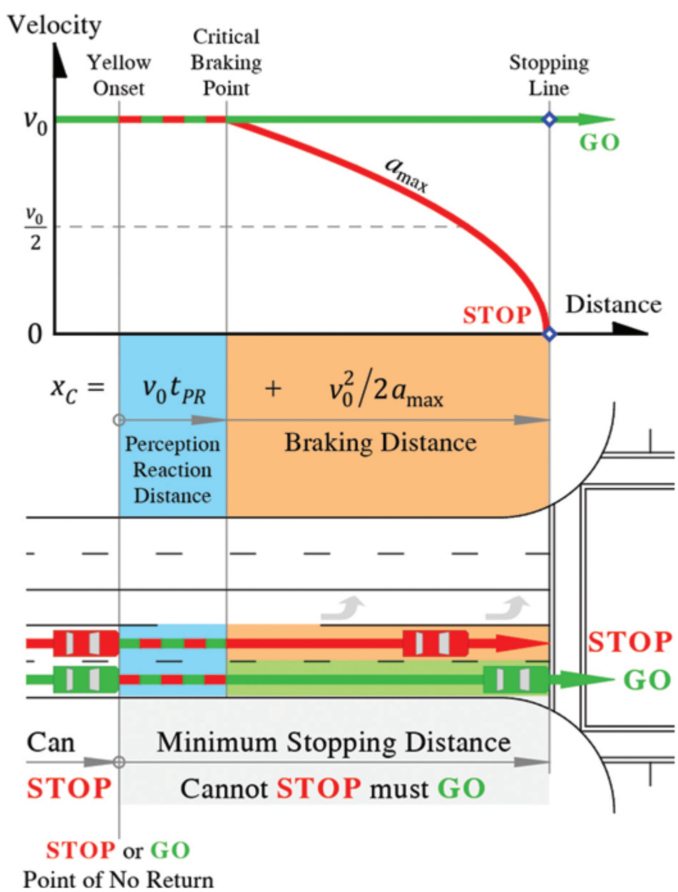

Figure 2. Illustration of the STOP or GO scenario encountered when approaching a signalized intersection.

the stopping distance while decelerating from the initial approach velocity (v_0) to the intersection entry velocity (v_E) to safely and comfortably negotiate a turning maneuver.

In contrast to the condition where a driver approaches a signalized intersection in a through lane, scenarios where a driver approaches a signalized intersection in a turning lane are significantly more complicated. Although there is a range of possibilities as to where a driver might begin to decelerate on approach to the intersection, the extended solution presented in this article is based on a model of driver-vehicle motion which encompasses the "worst-case scenario" or "boundary condition" for a decelerating vehicle. A full explanation of this concept and examination of other models of driver-vehicle motion is presented in "Yellow Change Intervals for Turning Movements Using Basic Kinematic Principles," available on the ITE website at www.ite.org/technical-resources/topics/traffic-engineering/traffic-signal-change-and-clearance-intervals.

Järlström's Extended Kinematic Equation

For the extended solution, conceive that the driver begins their deceleration at the Critical Braking Point, decelerating at their maximum safe and comfortable deceleration (a_{max}) to their target entry velocity (v_E) and then traverses the remainder of the braking distance at this velocity into the intersection.

Under this "boundary condition" model for a decelerating vehicle, the minimum stopping distance (x_C) is divided into three distinct areas of vehicle movement: 1) the Perception-Reaction zone (x_{PR}), 2) a Deceleration Zone (x_{Dec}) where the driver decelerates to their target entry velocity (v_E) beginning at the Critical Braking Point, and 3) a Non-Deceleration "Go Zone" (x_{Go}) starting at the end of the Deceleration Zone where the driver continues at their target entry speed to the limit line and into the intersection. Figure 3 illustrates these concepts.

The minimum time to traverse the minimum stopping distance is, therefore, the combination of 1) the time to traverse the perception-reaction distance (t_{PR}), plus 2) the time to traverse the Deceleration Zone (t_{Dec}), plus 3) the time to traverse the Go Zone (t_{Go}). This combination is the minimum yellow change interval (Y_{min}) necessary to eliminate the dilemma zone for this model of driver-vehicle motion, expressed as:

$$Y_{min} = t_{PR} + t_{Dec} + t_{Go} \tag{5}$$

The time to traverse the Deceleration Zone is given by:

$$t_{Dec} = \frac{(v_0 - v_E)}{a_{max}} \tag{6}$$

The time to traverse the Go Zone (t_{Go}) is determined as follows:

First, calculate the length of the Go Zone (x_{Go}) by subtracting the length of the Deceleration Zone (x_{Dec}) from the full braking distance (x_{Br}).

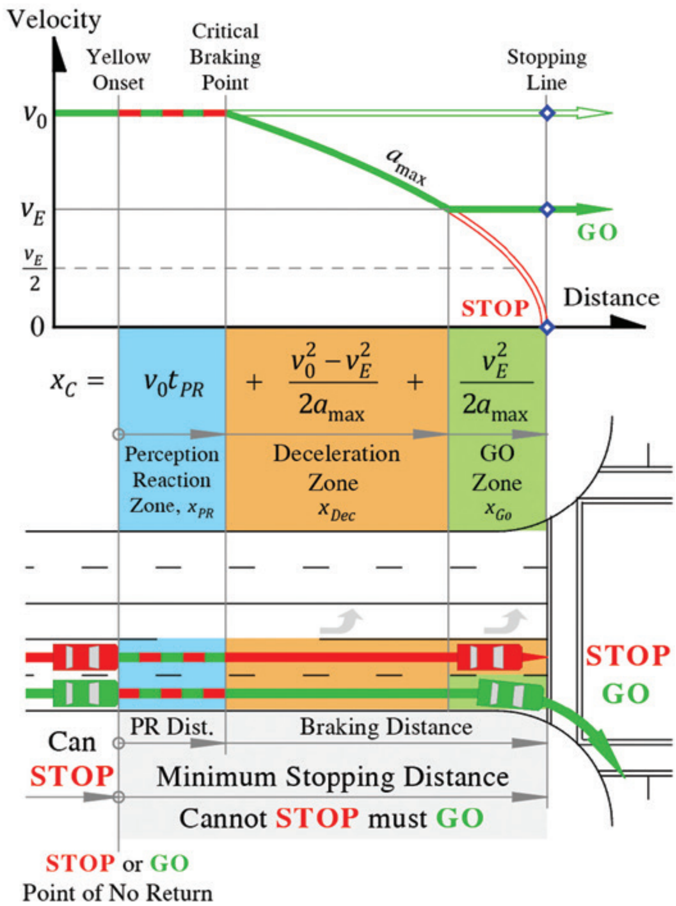

Figure 3. Zones of driver-vehicle motion while decelerating to negotiate a turn.

Since the length of the Deceleration Zone (x_{Dec}) equals the vehicle's time to traverse the Deceleration Zone (t_{Dec}) multiplied by the vehicle's average velocity (v_{av}):

$$x_{Dec} = v_{av} t_{Dec} = \frac{(v_0 + v_E)}{2} \bullet \frac{(v_0 - v_E)}{a_{max}} = \frac{v_0^2 - v_E^2}{2a_{max}} \quad (7)$$

And, from the last term of Equation 1, the braking distance is:

$$x_{Br} = \frac{v_0^2}{2a_{max}} \quad (8)$$

The length of the Go Zone is:

$$x_{Go} = x_{Br} - x_{Dec} = \frac{v_0^2}{2a_{max}} - \frac{v_0^2 - v_E^2}{2a_{max}} = \frac{v_E^2}{2a_{max}} \quad (9)$$

The time to traverse the Go Zone (t_{go}) equals the length of the Go Zone (x_{go}) divided by the vehicle's velocity across this distance (the driver's target entry velocity (v_E)):

$$t_{Go} = \frac{x_{Go}}{v_E} = \frac{\frac{v_E^2}{2a_{max}}}{v_E} = \frac{v_E}{2a_{max}} \quad (10)$$

Therefore, the minimum time to traverse the minimum stopping distance (by definition, the minimum yellow change interval, Y_{min}) for a vehicle that decelerates within the critical distance to negotiate a turn is given by:

$$Y_{min} = t_{PR} + \frac{(v_0 - v_E)}{a_{max}} + \frac{v_E}{2a_{max}} \quad (11)$$

Algebraic simplification of the Järlström's extended kinematic model shown in Equation 11 yields:

$$Y_{min} = t_{PR} + \frac{v_0 - ½v_E}{a_{max}} \quad (12)$$

Where ($v_0 \geq v_E > 0$):
Y_{min} = Minimum yellow change interval (s)
v_0 = Maximum uniform initial/approach velocity, (ft./s or m/s)
v_E = Maximum intersection entry velocity, (ft./s or m/s)
t_{PR} = Maximum allocated driver-vehicle perception-reaction time, (s)
a_{max} = Maximum uniform safe and comfortable deceleration, (ft./s^2 or m/s^2)

Figure 4 illustrates the extended kinematic model compared to GHM's STOP or GO solutions across the critical distance (x_C) referenced to time.

The validity of Järlström's Extended Kinematic Equation is established in the following manner:

When $v_E = v_0$ (constant velocity), the protocol yields the ITE Kinematic Equation applicable for through movements (Equation 3).

When $v_E = 0$ (zero end velocity), the protocol yields the equation to calculate the minimum time to come to a complete stop:

$$t_{Stop} = t_{PR} + \frac{v_0}{a_{max}} \quad (13)$$

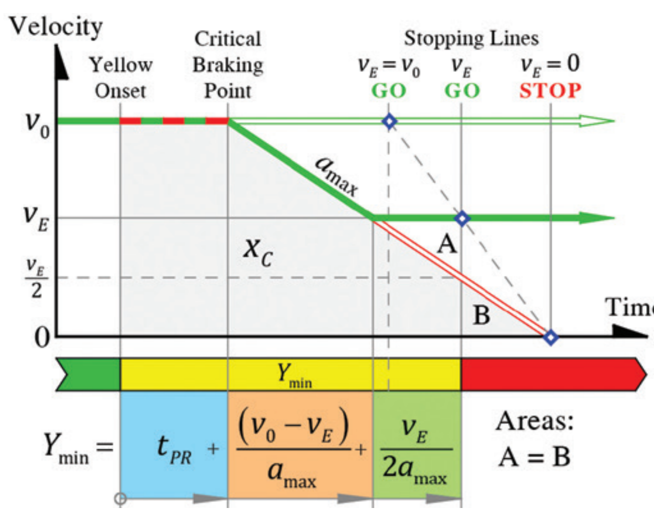

Figure 4. Time model including vehicle deceleration traversing the minimum stopping distance.

Note that stopping vehicles will reach the limit line after the signal has changed to red and, for these vehicles, the length of the yellow interval is irrelevant.

Additional Considerations

1. The methodology for determining the length of the yellow change interval described by both the classic and extended kinematic equations incorporates the following presumptions:
   a) The vehicle travels in free-flow conditions (unimpeded movement, no queue, etc.).
   b) The yellow indication illuminates at the moment the vehicle arrives at the critical distance.
   c) When the yellow illuminates, the vehicle's initial approach velocity (v_0) is the actual or estimated 85th percentile speed or the posted limit, whichever is higher.
2. The extended kinematic equation presented here yields the minimum yellow interval for a level intersection approach. As with the kinematic equation for through movements, grade adjustments should be made for vehicles approaching on a downgrade.
3. The assumed intersection entry velocity should be determined using engineering judgment. Generally, drivers entering an intersection to conduct a left turn, do so at approximately 20 miles per hour (mph) (32 kilometers per hour [km/hr]) depending on the intersection radius. Right-turning drivers generally negotiate the turn at approximately 12 mph (19 km/hr). An entry speed can also be estimated based on the *curve design speed* published by ITE.[6] For a full explanation of this calculation, see "Yellow Change Intervals for Turning Movements Using Basic Kinematic Principles," available at www.ite.org/technical-resources/topics/traffic-engineering/traffic-signal-change-and-clearance-intervals.
4. Calculating tolerance is standard engineering practice and should be employed in calculations of the minimum yellow change interval. Perception-reaction time, deceleration, approach velocity, and entry velocity are not constants. A reasonable range of values for each of these parameters is applicable for every driver-vehicle complex approaching a signalized intersection. Driver-vehicles whose metrics fall within a reasonable range but do not strictly match the parameters typically chosen by the traffic engineer should be accommodated.

   For example, research shows that the 85th percentile PRT is closer to 1.5 seconds (sec.) rather than the traditionally accepted PRT of 1.0 sec.[7] Likewise, some drivers, as well as larger vehicles, cannot safely and comfortably decelerate at 10 ft./s^2 (3.05 m/s^2) and employ a deceleration of 8.0 ft./s^2 (2.44 m/s^2) or less.[8] Therefore, engineering tolerances should be employed within signal timing protocols to accommodate all reasonable driver-vehicle combinations, especially where the rate of red-light violations is higher than acceptable.
5. The benefit of the extended kinematic equation is to provide a sufficient yellow change interval for all driver-vehicle movements to eliminate the dilemma zone and reduce red-light violations. Practitioners should be aware that red-light violations may increase in turning lanes if the available green time is reduced to accommodate longer yellow intervals. This is especially true where the green interval is insufficient to clear the queue. Rather than reducing the green interval, practitioners may consider increasing the cycle length instead.
6. Practitioners may have concerns about yellow intervals that are "excessive," resulting in drivers stopped at the signal still viewing a yellow indication. However, yellow intervals calculated using the extended solution do not exceed the minimum time required for a vehicle to come to a safe and comfortable STOP (Equation 13). Therefore the circumstance of a stopped driver facing a stale yellow light should typically not occur. **itej**

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